International Journal of Theoretical Physics, Vol. 33, No. 6, 1994

Čerenkov Radiation at Finite Temperature

Zdeněk Kopecký[1]

Received February 5, 1993

A power formula for Čerenkov radiation at finite temperature is derived in the framework of the generalized finite-temperature Cutkosky rules. Spins 1/2 and 0 are considered.

1. INTRODUCTION

Reactions rate for quantum processes taking place in a heat bath in thermal equilibrium have been actively studied the past few years. Computations of discontinuities at finite temperature and their physical interpretations in the framework of the imaginary-time finite-temperature field theory (IT FTFT) were done by Weldon (1983).

The generalization of Cutkosky rules in the real-time FTFT (the circled diagrams algorithm) was found in Kobes and Semenoff (1986). Recently, Niégawa (1990) and Ashida *et al.* (1991) showed that any Kobes–Semenoff diagram can be cut. Therefore the total discontinuity at finite temperature is a collection of different reaction rates. We use the result of this theory for concrete physical situations.

In this article we derive a spectral formula for finite-temperature Čerenkov radiation, i.e., the energy loss of a charged particle moving faster than the speed of light in the medium. We discuss quantum particles (spin 1/2 and 0) and we consider that the medium is filled by equilibrium photon radiation (finite-temperature situation).

The energy loss per unit time of the particle is defined by (Tzytovich, 1962)

$$-\frac{dE}{dt}=\int d\omega\,\omega P(\omega)$$
$$=\int d\omega\,\omega\{[N_B(\omega)+1]\Gamma^+(\omega)-N_B(\omega)\Gamma^-(\omega)\} \qquad (1)$$

[1]Department of Theoretical Physics and Astrophysics, Faculty of Science, Masaryk University, Kotlářská 2, 611 37 Brno, Czech Republic.

0020-7748/94/0600-1251$07.00/0

Here $P(\omega)$ is the spectral power distribution, and $\Gamma^{+}(\omega)$ and $\Gamma^{-}(\omega)$ are the rates for emission and absorption of one photon with energy ω, respectively. $N_B(\omega) = 1/[\exp(\beta\omega) - 1]$ is the Bose–Einstein distribution function, and β is the inverse temperature.

2. FINITE-TEMPERATURE DECAY RATE

The starting point for our calculation is the decay forward amplitude (Niégawa, 1990; Ashida *et al.*, 1991):

$$\Gamma_d(E) = \frac{-i}{2E}\Sigma_{21} \tag{2}$$

Σ_{21} is the off-diagonal on-shell RT FTFT self-energy (see Fig. 1).

There is a detailed discussion of the amplitude in Niégawa (1990) and Ashida *et al.* (1991). We assume the first-level diagram. There are propagators in Fig. 1, where the circled diagram convention is used (Kobes and Semenoff, 1986; Niégawa, 1990; Ashida *et al.*, 1991):

(a) Photon:

$$iD^{+\mu\nu}(k) = 2\pi\mu[\Theta(k^0) + N_B(|k^0|)][-g^{\mu\nu} + (1 - n^{-2})\eta^\mu\eta^\nu]\,\delta(n^2(k^0)^2 - (\mathbf{k})^2) \tag{3}$$

where $\eta^\mu = (1, \mathbf{0})$ and n is the index of refraction of the medium and μ is its magnetic permeability.

(b) Charged particles: We assume nonthermal propagators, which corresponds to single charged particle moving in the medium:

Spin 1/2:

$$iS^{+}(k) = 2\pi(m + \gamma k)\Theta(k^0)\delta(k^2 - m^2) \tag{4}$$

Spin 0:

$$iG^{+}(k) = 2\pi\Theta(k^0)\delta(k^2 - m^2) \tag{5}$$

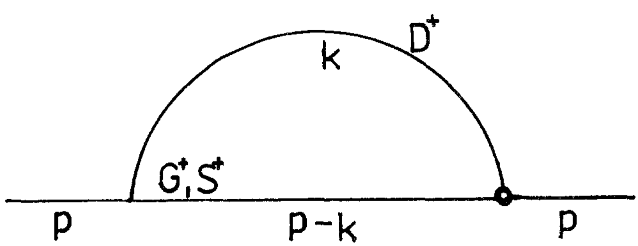

Fig. 1

The decay rate (2) is equal (in the case of spin one-half we take the average over the particle wave function) to the sum of $\int d\omega\, \Gamma^+(\omega)[N_B(\omega)+1]$ and $\int d\omega\, \Gamma^-(\omega)N_B(\omega)$ (Niégawa, 1990; Ashida *et al.*, 1991). We shall modify (2) by insertion of the energy loss of the particle into the self-energy diagram (2) to obtain formula (1):

In case of spin 1/2

$$iS^+(p) \mapsto (E-p^0)iS^+(p) \tag{6}$$

and in case of spin 0

$$iG^+(p) \mapsto (E-p^0)iG^+(p) \tag{7}$$

where E denotes energy of the charged particle and $E-p^0$ is its energy loss after one process of radiation.

3. SPIN ONE-HALF

The total spin-1/2-particle energy loss $-dE/dt$, equation (1), using (2), (6), will be

$$\begin{aligned} -\frac{dE}{dt} &= \int_0^\infty d\omega\, \omega P(\omega) \\ &= \frac{1}{2E}\left\langle e^2 \int \frac{d^4k}{(2\pi)^4}\, \gamma^\mu k^0 iS^+(p-k)\gamma^\nu iD^+_{\mu\nu}(k)\right\rangle \end{aligned} \tag{8}$$

$\langle\cdot\rangle$ denotes the average over spin states $\sum_\sigma \bar{u}(p,\sigma)\ldots u(p,\sigma)$, where $u(p,\sigma)$ is the particle wave function normalized by the condition $\langle 1\rangle = 2m$, where m is the mass of the particle. Equation (8) takes the form, after substitution (3), (4)

$$\begin{aligned} -\frac{dE}{dt} = \frac{e^2\mu}{2E}\int \frac{d^4k}{(2\pi)^4}\, T(p,k)k^0[\Theta(k^0)+N_B(k)]\Theta(p^0-k^0) \\ \times \delta(n^2(k^0)^2-(\mathbf{k})^2)\delta((p-k)^2-m^2) \end{aligned} \tag{9}$$

Containing all spin operations in $T(p,k)$,

$$T(p,k) = \langle \gamma^\mu[m+\gamma(p-k)][-g_{\mu\nu}+(1-n^{-2})\eta_\mu\eta_\nu]\gamma^\nu\rangle \tag{10}$$

After spin reduction in (10) we obtain

$$T(p,k) = -2m + 2(1-n^{-2})\frac{E^2}{m} - k^0\frac{E}{m}(3-n^{-2}) + (1+n^{-2})\frac{\mathbf{kp}}{m} \tag{11}$$

We make standard operations for the δ-function in (4):

$$\delta((p-k)^2-m^2)=\frac{\delta(p^0-k^0+E_{\mathbf{p}-\mathbf{k}})+\delta(p^0-k^0-E_{\mathbf{p}-\mathbf{k}})}{2E_{\mathbf{p}-\mathbf{k}}}$$

$$\delta(n^2(k^0)^2-(\mathbf{k})^2)=\frac{\delta(k^0-|\mathbf{k}|/n)}{2E_{\mathbf{k}}}+\frac{\delta(k^0+|\mathbf{k}|/n)}{2E_{\mathbf{k}}}$$

$$E_{\mathbf{k}}=\frac{|\mathbf{k}|}{n},\qquad E_{\mathbf{p}-\mathbf{k}}=[m^2+(\mathbf{p}-\mathbf{k})^2]^{1/2} \tag{12}$$

After substution of (12) in (9) and transforming the momentum measure d^4k to a more convenient form, we get

$$\begin{aligned}-\frac{dE}{dt}=e^2\mu\int\frac{d\omega\, d(\cos(\theta))}{8\pi vE}\,\omega T(p,k)\\ \times\{\delta(\cos(\theta)-\cos(\theta_e))[1+N_B(\omega)]\\ -\delta(\cos(\theta)+\cos(\theta_a))N_B(\omega)\}\end{aligned} \tag{13}$$

where

$$\begin{aligned}\cos(\theta_e)=\frac{1+(n^2-1)\omega/2E}{nv},\qquad \cos(\theta_a)=\frac{1-(n^2-1)\omega/2E}{nv}\\ v=\frac{|\mathbf{P}|}{E},\qquad \omega=\frac{|\mathbf{k}|}{n},\qquad \cos(\theta)=\frac{\mathbf{kp}}{|\mathbf{k}|\cdot|\mathbf{p}|}\end{aligned} \tag{14}$$

Now we can complete the calculations. Substitution of $T(\omega,\cos(\theta),E)$ from (11) into (13) and trivial integration over $d(\cos(\theta))$ give the final result for the energy loss:

$$\begin{aligned}-\frac{dE}{dt}&=\int_0^\infty d\omega\,\omega P(\omega)\\ &=\int_{\cos(\theta_e)<1}d\omega\,\omega\frac{e^2\mu}{4\pi}v\left\{1-\frac{1}{n^2v^2}\left[1+\frac{\omega}{E}(n^2-1)-\frac{\omega^2}{4E^2}(n^4-1)\right]\right\}\\ &\quad\times[N_B(\omega)+1]-\int_{|\cos(\theta_a)|<1}d\omega\,\omega\frac{e^2\mu}{4\pi}v\\ &\quad\times\left\{1-\frac{1}{n^2v^2}\left[1-\frac{\omega}{E}(n^2-1)-\frac{\omega^2}{4E^2}(n^4-1)\right]\right\}N_B(\omega)\end{aligned} \tag{15}$$

where $\cos(\theta_e)$ and $\cos(\theta_a)$ are defined in (14) and $\omega>0$.

4. SPIN ZERO

In the case of spin 0 we substitute the propagator (7) for the charged particle and the standard vertex form for scalar field theory in the self-energy (2). Then the total energy loss $-dE/dt$ is

$$-\frac{dE}{dt} = \int_0^\infty d\omega\, \omega P(\omega)$$
$$= \frac{1}{2E} e^2 \int \frac{d^4k}{(2\pi)^4} k^0 (2p-k)^\mu (2p-k)^\nu iG^+(p-k) iD^+_{\mu\nu}(k) \quad (16)$$

After substitution of (3) and (5) in (16), we get

$$-\frac{dE}{dt} = \int_0^\infty d\omega\, \omega P(\omega)$$
$$= \int_{\cos(\theta_e)<1} d\omega\, \omega \frac{e^2\mu}{4\pi} v \left\{1 - \frac{1}{n^2v^2}\left[1 + \frac{\omega}{E}(n^2-1) - \frac{\omega^2}{8E^2}(n^2-1)^2\right]\right\}$$
$$\times [N_B(\omega)+1] - \int_{|\cos(\theta_a)|<1} d\omega\, \omega \frac{e^2\mu}{4\pi} v$$
$$\times \left\{1 - \frac{1}{n^2v^2}\left[1 - \frac{\omega}{E}(n^2-1) - \frac{\omega^2}{8E^2}(n^2-1)^2\right]\right\} N_B(\omega) \quad (17)$$

where $\cos(\theta_e)$ and $\cos(\theta_a)$ are defined in (14) and $\omega > 0$.

5. CLASSICAL LIMIT

In both cases spin 1/2 and 0 the spectral function $P(\omega)$ has in the classical limit $\omega/E \mapsto 0$ the same form:

$$\begin{aligned} nv > 1: &\quad P(\omega) = \frac{e^2\mu}{4\pi} v\left(1 - \frac{1}{n^2v^2}\right) \\ nv < 1: &\quad P(\omega) = 0 \end{aligned} \quad (18)$$

Therefore in the classical case there is no temperature-dependent contribution to the radiation as found in Kirzhnitz (1990).

6. DISCUSSION

We have used an elegant method of the generalized finite-temperature Cutkosky rules to derive a power formula for the Čerenkov radiation at finite temperature. Our results in the case of spin 1/2 are similar to Tzytovich (1962), where the old one-component real-time formalism was used. We are also in accordance with the result of Kirzhnitz (1990), where the general method for calculations of energy loss is derived from the fluctuation-dissipation theorem.

The result of Pardy (1989), according to our calculations, is the sum of the absorption and radiation power in the spectral formula.

ACKNOWLEGMENTS

I would like to thank Dr. M. Pardy for discussion. I thank also Dr. A. Niégawa for sending me his papers.

REFERENCES

Ashida, A., Nakkagawa, H., Niégawa, A., and Yokota, H. (1991). Osaka City University preprints OCU-137 and OCU-138.

Kirzhnitz, D. A. (1990). *Zhurnal Experimentalnoi i Teoreticheskoi Fiziki*, **97**, 1089.

Kobes, R. L., and Semenoff, G. W. (1986). *Nuclear Physics B*, **260**, 714; **272**, 329.

Niégawa, A. (1990). *Physics Letters*, **247B**, 351.

Pardy, M. (1989). *Physics Letters*, **134A**, 357.

Tzytovich, V. N. (1962). *Zhurnal Experimentalnoi i Teoreticheskoi Fiziki*, **42**, 803.

Weldon, H. A. (1983). *Physical Review D*, **28**, 2007.